General Certificate of Education
Advanced Subsidiary (AS) and Advanced Level

MATHEMATICS M2

Mechanics 2

Additional materials:
Answer paper
Graph paper
List of Formulae

SPECIMEN PAPER

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.

Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.

You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

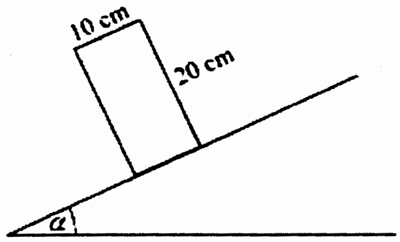
The total number of marks for this paper is 60.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

1



The diagram shows the cross-section of a uniform solid rectangular block. This cross-section has dimensions 20 cm by 10 cm and lies in a vertical plane. The block rests in equilibrium on a rough plane whose inclination α to the horizontal can be varied. The coefficient of friction between the block and the plane is 0.7. Given that α is slowly increased from zero, determine whether equilibrium is broken by toppling or sliding. **[5]**

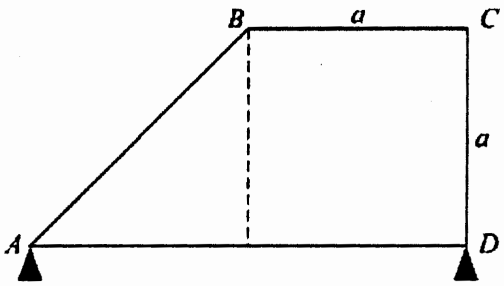
2 A small ball of mass 0.2 kg is dropped from rest at a height of 1.2 m above a horizontal floor. The ball rebounds vertically from the floor, reaching a height of 0.8 m. Assuming that air resistance can be neglected, calculate

(i) the coefficient of restitution between the ball and the floor, **[4]**

(ii) the impulse exerted by the floor on the ball when the ball bounces. **[2]**

If air resistance were taken into account, would the value calculated for the coefficient of restitution be larger or smaller than the value calculated in part (i)? Justify your conclusion. **[1]**

3



A uniform lamina *ABCD* has the shape of a square of side a adjoining a right-angled isosceles triangle whose equal sides are also of length a. The weight of the lamina is W. The lamina rests, in a vertical plane, on smooth supports at A and D, with AD horizontal (see diagram).

(i) Show that the centre of mass of the lamina is at a horizontal distance of $\frac{11}{9}a$ from A. **[4]**

(ii) Find, in terms of W, the magnitudes of the forces on the supports at A and D. **[4]**

4

Fig. 1

Fig. 1 shows the cross-section of a hollow container. The base of the container is circular, and is horizontal. The sloping part of the side makes an angle of 15° with the horizontal, and the vertical part of the side forms a circular cylinder of radius 0.4 m. A small steel ball of mass 0.1 kg moves in a horizontal circle inside the container, in contact with the vertical and sloping parts of the side at A and B respectively, as shown in Fig. 2.

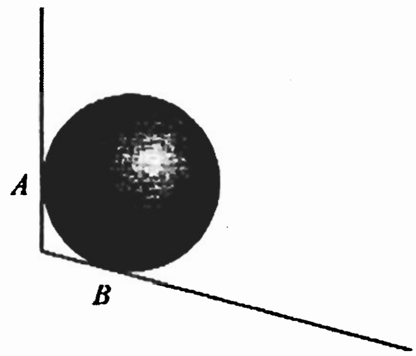


Fig. 2

It is assumed that all contacts are smooth and that the radius of the ball is negligible compared to 0.4 m.

(i) Given that the ball is moving with constant speed 3 m s^{-1}, find the magnitudes of the contact forces acting on the ball at A and at B. **[5]**

(ii) Calculate the least speed that the ball can have while remaining in contact with the vertical part of the side of the container. **[3]**

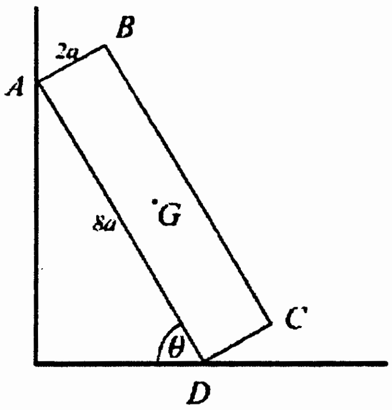
5 A car of mass 650 kg is travelling on a straight road which is inclined to the horizontal at 5°. At a certain point P on the road the car's speed is 15 m s^{-1}. The point Q is 400 m down the hill from P, and at Q the car's speed is 35 m s^{-1}.

(i) Assume that the car's engine produces a constant driving force on the car as it moves down the hill from P to Q, and that any resistances to the car's motion may be neglected. By considering the change in energy of the car, or otherwise, calculate the magnitude of the driving force of the car's engine. **[4]**

(ii) Assume instead that resistance to the car's motion between P and Q may be represented by a constant force of magnitude 900 N. Given that the acceleration of the car at Q is zero, show that the power of the car's engine at this instant is approximately 12.1 kW. **[4]**

Given that the power of the car's engine is the same when the car is at P as it is when the car is at Q, calculate the car's acceleration at P. **[2]**

6



A uniform rectangular box of weight W stands on a horizontal floor and leans against a vertical wall. The diagram shows the vertical cross-section $ABCD$ containing the centre of mass G of the box. AD makes an angle θ with the horizontal, and the lengths of AB and AD are $2a$ and $8a$ respectively.

(i) By splitting the weight into components parallel and perpendicular to AD, or otherwise, show that the anticlockwise moment of the weight about the point D is $Wa(4\cos\theta - \sin\theta)$. **[3]**

(ii) The contact at A between the box and the wall is smooth. Find, in terms of W and θ, the magnitude of the force acting on the box at A. **[3]**

(iii) The contact at D between the box and the ground is rough, with coefficient of friction μ. Given that the box is about to slip, show that $\tan\theta = \dfrac{4}{8\mu + 1}$. **[4]**

7

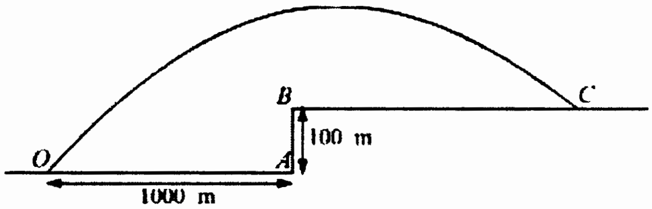


Fig. 1

A shell is fired from a stationary ship O which is at a distance of 1000 m from the foot of a vertical cliff AB of height 100 m. The shell passes vertically above B and lands at a point C on horizontal ground, level with the top of the cliff (see Fig. 1). The shell is fired with speed 300 m s^{-1} at angle of elevation θ, and air resistance to the motion of the shell may be neglected.

(i) Given that $\theta = 30°$, find the time of flight of the shell and the distance BC. **[6]**

(ii)

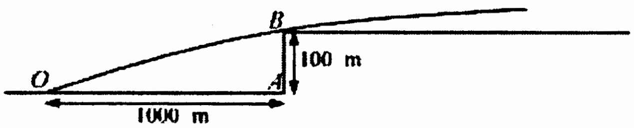


Fig. 2

Given instead that the shell *just* passes over B, as shown in Fig. 2, find the value of θ, correct to the nearest degree. **[6]**

1	Topples when CG is above lowest corner i.e. when $\tan\alpha = \frac{1}{2}$ Slides when $R = mg\cos\alpha$ and $0.7R = mg\sin\alpha$ i.e. when $\tan\alpha = 0.7$ Hence it topples, since $\frac{1}{2} < 0.7$	B1 B1 M1 A1 B1✓	 5	May be implied Both equations attempted Allow B2 in place of M1 A1 if $\mu = \tan\alpha$ is quoted Conclusion and reason needed
2	(i) Speed before impact is $\sqrt{2g\times1.2}$ and $\sqrt{2g\times0.8}$ Hence $e = \frac{\sqrt{1.6g}}{\sqrt{2.4g}} = \sqrt{\frac{2}{3}} \approx 0.816$	M1 A1 M1 A1	 4	For one relevant use energy or const acc For both correct (unsimplified) values For use of $v' = ev$ to calculate e For correct exact or decimal answer
	(ii) Impulse $= 0.2(\sqrt{2.4g} + \sqrt{1.6g}) \approx 1.76\,\text{N s}$	M1 A1✓	 2	Allow M mark even if there is a sign error
	With air resistance, speed before impact is smaller, and speed after impact is larger; hence e is larger	B1	1	For correct conclusion with correct reasons
3	(i) CG of triangle is $\frac{2}{3}a$ horizontally from A Moments: $\frac{1}{3}W\times\frac{2}{3}a + \frac{2}{3}W\times\frac{3}{2}a = W\times\bar{x}$ Hence $\bar{x} = \frac{11}{9}a$	B1 M1 A1 A1	 4	 For equating moments about A, or equivalent For a correct unsimplified equation Given answer correctly shown
	(ii) $R_A \times 2a = W \times \frac{7}{9}a \Rightarrow R_A = \frac{7}{18}W$ $R_A + R_D = W \Rightarrow R_D = \frac{11}{18}W$	M1 A1 M1 A1✓	 4	For one moments equation For one correct answer For resolving, or a second moments equation For a second correct answer
4	(i) $R_B\cos15° = 0.1g$ Hence $R_B = 1.01\,\text{N}$ $R_A + R_B\sin15° = 0.1\times\frac{3^2}{0.4}$ Hence $R_A = 1.99\,\text{N}$	B1 B1 M1 A1 A1	 5	 For using NII horizontally (3 terms needed) Correct equation
	(ii) $R_A = 0$, and $R_B = 1.01$ as before $R_B\sin15° = 0.1\times\frac{v^2}{0.4}$ $v = 1.02\,\text{m s}^{-1}$	B1✓ M1 A1	 3	May be implied

5	(i) *EITHER*: $\Delta KE = \frac{1}{2} \times 650 \times (35^2 - 15^2)$	B1		For correct unsimplified expression
	$\Delta PE = 650g \times 400 \sin 5°$	B1		Sign of change not required
	$400T = 325\,000 - 222\,073$	M1		For equating work and energy change
	$T = 257$ N	A1		
	OR: $a = \dfrac{35^2 - 15^2}{2 \times 400} = 1.25$	B1		For correct unsimplified expression for a
	$T + 650g \sin 5° = 650a$	M1		For 3-term NII equation ǁ slope
		A1		Correct equation
	$T = 257$ N	A1	4	
	(ii) Driving force at Q is $\dfrac{H}{35}$	B1		For any correct statement of '$P = Fv$'
	$900 = \dfrac{H}{35} + 650g \sin 5°$	M1		For 3-term force equation ǁ slope at Q
		A1		Correct equation
	Hence $H \approx 12069$ W i.e. 12.1 kW	A1	4	Given answer correctly shown
	$\dfrac{H}{15} + 650g \sin 5° - 900 = 650a$	M1		For 4-term NII equation
	$a = 0.707 \text{ ms}^{-2}$	A1	2	
6	(i) *EITHER*: Moments of weight components are:			
	$W \cos\theta \times 4a$ (anticlockwise)	B1		
	$W \sin\theta \times a$ (clockwise)	B1		
	Hence total anticlockwise is $Wa(4\cos\theta - \sin\theta)$	B1		Given answer correctly shown
	OR: Horizontal distance $G - D$ is	M1		For using horizontal projections
	$-a \sin\theta + 4a \cos\theta$	A1		Correct expression
	Hence anticlockwise moment is $Wa(4\cos\theta - \sin\theta)$	A1		Given answer correctly shown
	OR: Horizontal distance $G - D$ is $\sqrt{(4a)^2 + a^2} \cos(\theta + \alpha)$ where $\tan\alpha = \frac{1}{4}$	B1		
	Anticlockwise moment is $W \times \sqrt{17a^2}\left(\frac{4}{\sqrt{17}}\cos\theta - \frac{1}{\sqrt{17}}\sin\theta\right)$	M1		
	i.e. $Wa(4\cos\theta - \sin\theta)$	A1	3	Given answer correctly shown
	(ii) $Wa(4\cos\theta - \sin\theta) = R_A \times 8a \sin\theta$	M1		For moments equation about D, using **(i)**
		A1		Correct equation
	$R_A = \frac{1}{8}W(4\cot\theta - 1)$	A1	3	For correct answer, in any form
	(iii) $R_D = W$	B1		
	$F_D = R_A$	B1		
	$\mu = \dfrac{4\cot\theta - 1}{8}$	M1		For use of $F = \mu R$ in equation involving θ
	Hence $\tan\theta = \dfrac{4}{8\mu + 1}$	A1	4	For showing given result correctly

7 (i) *EITHER* : $100 = 150t - 4.9t^2$	M1	For use of $s = ut + \frac{1}{2}at^2$ vertically
	A1	For correct equation
$t = \dfrac{150 \pm \sqrt{20540}}{9.8}$	M1	For any solution method
Time to C is 29.9 s	A1	
$x = 300\cos 30° \times 29.9$	M1	
$BC = x - 1000 \approx 6780$ m	A1√	
OR : $100 = \dfrac{x}{\sqrt{3}} - \dfrac{9.8x^2}{2 \times 300^2 \times \frac{3}{4}}$	M1	For trajectory equation with $y = 100$, $V = 300$, $\theta = 30°$
	A1	For correct unsimplified equation
$x \approx 7776$	M1	For any solution method
Hence $BC \approx 6780$ m	A1	
$t = \dfrac{x}{300\cos 30°}$	M1	
$= 29.9$ s	A1√ 6	
(ii) $100 = 1000\tan\theta - \dfrac{9.8 \times 1000^2}{2 \times 300^2}(1 + \tan^2\theta)$	M1	For trajectory equation with $y = 100$, $x = 1000$, $V = 300$
	M1	For use of $\sec^2\theta = 1 + \tan^2\theta$
$49\tan^2\theta - 900\tan\theta + 139 = 0$	A1	Correct simplified quadratic
$\tan\theta = \dfrac{900 \pm \sqrt{900^2 - 27244}}{98}$	M1	For any solution method
	M1	For taking arctan of the smaller root
$\theta \approx 9°$	A1 6	